

LOCAL CHARACTERISTICS OF AN AXISYMMETRIC IMPACT JET

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We present the results of an experimental investigation of the local characteristics of flow in an axisymmetric impact jet.

The high intensity of heat and mass exchange processes when jets strike a surface determines their wide application in various branches of engineering. In a number of cases, the use of jets allows one to increase the efficiency of cooling or heating by a factor of 3–5 compared with immersion in an axial flow. This fact and the comparatively small energy expenditures for jets have stimulated a great amount of research to advance the understanding of the mechanisms underlying the enhancement of the processes of transfer when jets strike on a barrier [1–3]. There is an extensive literature concerning the laws governing heat and mass exchange in the neighborhood of the stagnation point. In a number of works it has been found that the maximum enhancement of transfer processes at the stagnation point was achieved at a distance of the wall from the nozzle H equal to 6–8 nozzle diameters d [4–7]. At the same time, the explanations of this phenomenon in the literature are very contradictory. In [4] the maximum of the Nusselt number Nu at the stagnation point is attributed to the superposition of two opposite tendencies in the development of a jet: the onset of axial velocity decay at $x/d = 5–6$ and the increase in the intensity of turbulence on the jet axis. On the other hand, in [8] the heat and mass exchange enhancement at the stagnation point at $H/d = 6–8$ is attributed to a periodic renewal of the surface by large-scale vortical structures.

In spite of a comprehensive bibliography, many of the questions associated with the structure of flow near a wall in a gradient flow region remain unexamined. In particular, such an important physical quantity as shear stress on the wall was considered only in rare publications. In [9] measurements were made by Preston surface tubes; baffle plate fittings were employed by Yudaev in [3], heat sensors were used in [10, 11], and an electric diffusion method was applied by Kataoka et al. in [8, 12]. None of the procedures used earlier allowed measurement of shear stress in the neighborhood of the stagnation point or estimation of the level of friction fluctuations, since large-size single-component transducers were used.

The present work is concerned with experimental investigation of the characteristics of flow in the gradient region of a submerged axisymmetric impact jet.

The flow scheme is presented in Fig. 1a. The working section is a part of a closed hydrodynamic loop. It consists of a nozzle unit 1 with changeable fittings 2 and a barrier 3 located normally to the nozzle axis. The barrier is either a disk with friction and pressure transducers located on it or an adjustable rectangular plate; it is fixed along the coordinate r with an accuracy of 0.1 mm. The working fluid is delivered by a centrifugal pump 4, the fluid flow rate is controlled by a system of float rotameters 5.

To determine the hydrodynamic characteristics, an electrodiffusion method was applied in which the rate of electrochemical reaction occurring in the cathode (transducer)–electrolyte–anode system was measured [13, 14]. With a dc voltage between the cathode and anode, a current appears in the circuit whose magnitude in the diffusional regime is associated with a certain hydrodynamic quantity, depending on the design of the transducer.

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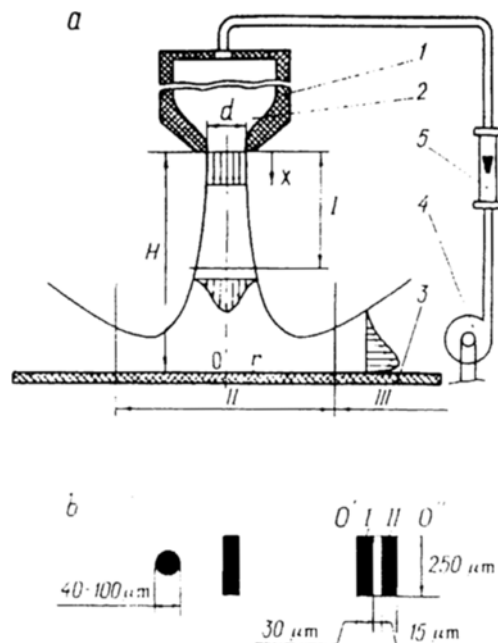


Fig. 1. Schematic diagram of experimental setup (a) and construction of the transducers of shear stress on the wall (b).

To measure fluid velocity, we used two types of transducers in the present work. The first is a "stagnation point" transducer, which is a thin platinum wire $40-50 \mu\text{m}$ in diameter sealed in a glass capillary. The sensitive element of this transducer is the polished end face of the platinum wire directed opposite to the flow. The outer diameter of the transducer does not exceed $70-80 \mu\text{m}$. The transducer of the second type has a sensitive element in the form of a rectangular platinum plate of $50 \times 600 \mu\text{m}$. The plate-electrode is mounted flush with the side surface of a glass wedge at a distance of about 1.5 millimeters from its edge [13-15].

The transducers of the shear stress on the wall have the shape and dimensions shown in Fig. 1b. A platinum wire or plate is sealed in a glass tube, which is then glued into the wall and ground flush with the surface.

Calculation of the mass transfer to the electrode in the vicinity of the stagnation point for each of the designs of velocity transducers establishes a correlation between the transducer current i in the diffusional regime and the flow velocity u : $i = cu^{1/2}$. For the transducers of shear stress τ , assuming a linear velocity profile near the wall, the relation has the form: $\tau = ai^3$, where the coefficients c and a depend on the size of the transducer, the shape of its nose, the viscosity and diffusion coefficients, and the number of ions participating in the reaction and their concentration in the solution. With microelectrodes, it is very difficult to apply design formulas, because of the large error in determining their dimensions and shape factor; therefore, a relative version of an electrodiffusion method was used. The velocity transducers were calibrated by the shock velocity profile at the tip of a well-profiled nozzle.

The shear stress transducers were calibrated by two independent techniques. The first consisted in the use of an exact solution for a laminar spatial flow in the neighborhood of the stagnation point. In this case the transducer was located in the region of linear variation of friction. The second technique was based on the solution of a momentum equation for a laminar axisymmetric boundary layer using for closure the measured distributions of static pressure, which, starting from $H/d = 5$, were approximated by the self-similar relation

$$P/P_0 = [1 + 0.757 (r/b_{0.5})^2]^{-4},$$

where $b_{0.5}$ is the half-width of the pressure profile. In this case, calibration was performed at the point of maximum friction; this corresponded to a distance of $(0.7-0.8)r/d$ from the stagnation point.

A fundamental feature of the present work was the use of a double electrochemical transducer to measure the average and fluctuating values of friction. Such a transducer allows one at each time instant to measure the

friction vector, i.e., its absolute value and direction. The transducer is made of two platinum plates of $30 \times 250 \mu\text{m}$ sealed in a glass envelope at a distance of $15\text{--}20 \mu\text{m}$ from each other (Fig. 1b). The principle of its operation is as follows. If fluid moves in the $O'O''$ direction, then electrode II turns out to be in the diffusion wake of electrode I, which is depleted of active ions, and the current in its circuit will be smaller than in the circuit of the first electrode. In this case, the amplification factors of the electrodiffusion transducers are set first so that the levels of the signals from both electrodes with independent switching are identical. An electronic circuit compares the signals from the transducers and sends to the output a signal from the first electrode along the stream with the corresponding sign. Thereafter, this alternating-sign signal is processed by a personal computer.

However, the double electrochemical friction transducer, suggested for the first time in [16, 17], had a limited application for measurements in complex flows. One of the main reasons is the difficulty of producing two identical electrodes separated by a very narrow gap (smaller than $20 \mu\text{m}$). Another reason is associated with the complexity of analysis of the signal from such a transducer in an alternating-sign flow.

Measurements of the average static pressure were made using samplers on a movable wall with recording by a standard "Safir" transducer.

To diagnose pressure fluctuations, piezoceramic transformers were used [15]. As a sensitive element, a ZTP-19 piezoelectric ceramic was used in the form of a cylinder 15 mm long and 1.5 mm in diameter. The diameter of the receiving surface of the transducer was about 1 mm. For amplitude-frequency calibration of the piezoelectric transducer, a device with a breaking diaphragm was employed that made it possible to obtain an input signal in the form of a step function. In this case the amplitude-frequency characteristic was calculated from the relation

$$H(\omega) = \int_0^{\infty} U(t) \exp(i\omega t) dt / \int_0^{\infty} S(t) \exp(i\omega t) dt,$$

where $U(t)$ is the response of the transducer to the input step function $S(t)$.

Correspondingly, the correction of the output signal of the transducer could be made according to the formula

$$Q(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) \Phi(\omega) \exp(-i\omega t) d\omega,$$

where $Q(t)$ is the response of the meter to a real input signal, $\Phi(\omega)$ are the Fourier components of the input signal.

The generally adopted scheme of flow of an impact jet presupposes the arbitrary division of the flow into three main regions (Fig. 1a): I, a region of a free turbulent jet; III, a region of jet wall flow; and II, a transition (gradient) region. All the relations for free and wall jets remain valid in the first and third regions, respectively. The gradient flow region is of main interest for investigating physical characteristics due to a nonmonotonic change in the flow parameters and to a considerable enhancement of heat and mass exchange processes.

In conformity with the literature data, the flow development in the gradient and wall zones is influenced by the free turbulent jet parameters in region I. Therefore, we conducted measurements of the average and fluctuational components of the longitudinal velocity components in a jet flowing out of nozzles of different shapes in the range of distances from the nozzle tip $2 \leq x/d \leq 10$. This range embraces the initial, transition, and the main portions of flow of the free turbulent jet. It is found that the distributions of both the average and fluctuational velocities are almost identical for different well-profiled contracting nozzles. At small distances from the nozzle tip we observed a distinct potential core with a low level of fluctuations. The distributions of the fluctuational velocity across the jet have a local minimum on the axis and a maximum on the jet boundary. As the distance x increases, the profile of the longitudinal fluctuational velocity levels out, and the maximum shifts gradually toward the flow axis. The averaged velocity profiles become self-similar starting with $x/d > 5\text{--}6$. Nozzles with a biradial generatrix (Vitoshinsky profile), an elliptic convex generatrix, or conical shape with reduction to a cylindrical shape near the outlet edge are well-profiled nozzles.

When a jet flows out of a poorly profiled nozzles (a cylindrical nozzle or a confuser nozzle with imitation of different degrees of wear) self-similarity appears in the distributions of velocity already at $x/d = 3$. In this case

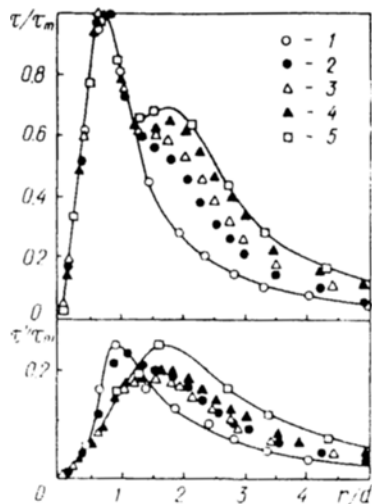


Fig. 2. Distributions of average and fluctuational values of friction on wall, $H/d = 3$: 1-4) $d = 4.2$ mm; 5) $d = 8$ mm; 1) $Re = 8800$; 2) 20,000; 3) 25,000; 4) 40,000, 5) 52,000.

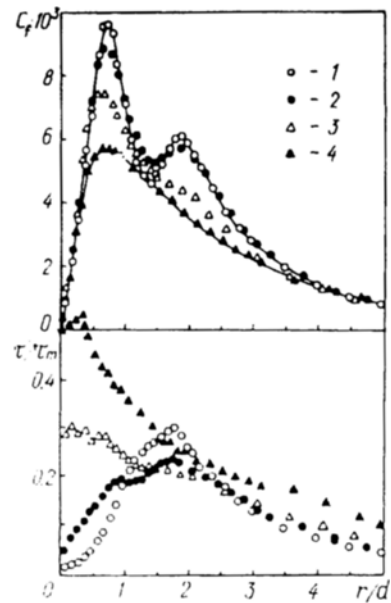


Fig. 3. Distributions of friction factor and root-mean-square fluctuations of friction on wall, $Re = 41,600$, $d = 10$ mm: 1) $H/d = 2$; 2) 4; 3) 6; 4) 8.

the level of velocity fluctuations turns out to be much higher, and its minimum on the flow axis is not distinct. Away from the nozzle tip an increased degree of liquid ejection is observed, as indicated by a large angle of jet expansion. For a cylindrical nozzle, the law of jet expansion has the form $r/x \cong 0.17$ while for well profiled nozzles all the data are satisfactorily generalized by the well-known relation [1] $r/x \cong 0.097$.

The results obtained confirm the conclusion that the influence of initial conditions on the hydrodynamic characteristics of submerged jets flowing out of well-profiled nozzles can be appreciable only in the initial region. An exception occurs in cases of strong flow turbulization [18] or an acoustic effect on the jet.

Shear stresses on the barrier for an axisymmetric jet were measured in a wide range of Reynolds numbers Re ($8 \cdot 10^3 \leq Re \leq 7.5 \cdot 10^4$). We used well-profiled nozzles with a biradial generatrix and outlet diameter equal to $d = 4.2, 5.5, 8,$ and 10 mm. The average velocity profile at the nozzle outlet was close to the shock profile with a nonuniformity degree not exceeding $u_m/u_0 = 1.02$; the turbulence level did not exceed $T_{um} = 0.005-0.007$.

The distributions of shear stresses and their fluctuations along the barrier as functions of the Reynolds number of the jet are presented in Fig. 2 for $H/d = 3$. This corresponds to the case when the plane of the barrier is located in the initial region of the jet. In the neighborhood of the stagnation point we observe a linear increase in friction. At $r/d \approx 0.75-0.8$ its value attains a maximum and then decreases with an increase in r . However, the change in friction in this region depends on the Reynolds number of the impinging jet. At low Re numbers the decrease in friction is monotonic, while the maximum of the fluctuations of friction coincides with the maximum of mean friction. As Re increases (in the present work its values were varied by regulating the jet discharge velocity and using nozzles of different diameters) the monotonicity in the friction decrease along the barrier is violated, and a second local maximum appears. Its magnitude increases, and when $Re \geq (4-5) \cdot 10^4$ the distributions of the average shear stress on the wall become self-similar with respect to the Reynolds number. As Re increases, the maximum of friction fluctuations moves away from the stagnation point, and its position virtually coincides with the position of the second maximum of friction.

The evolution of the distributions of the friction factor $C_f = \tau / (\rho u_0^2 / 2)$ and of the level of friction fluctuations as functions of the distance between the nozzle tip and the plate are shown in Fig. 3. As H/d increases from 2 to 10, the character of the change in friction along the barrier changes: starting from $H/d = 6$ the double-humped

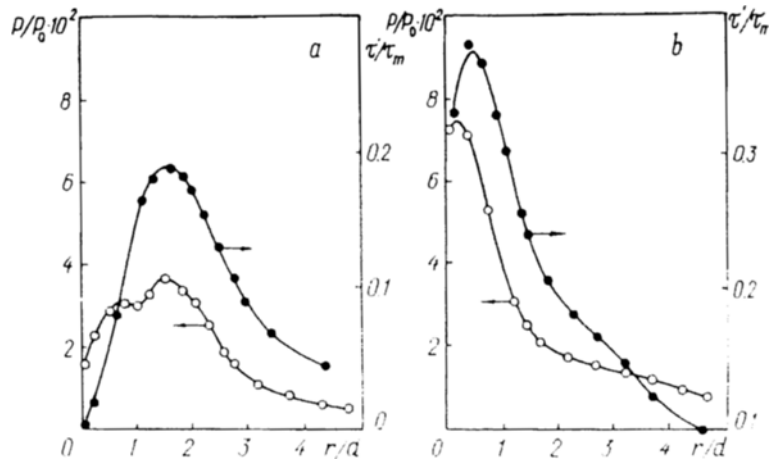


Fig. 4. Fluctuations of friction and pressure on barrier, $d = 4.2$ mm, $Re = 25,000$: a) $H/d = 3$; b) $H/d = 10$.

structure disappears. The distributions of the root-mean-square fluctuations of friction also change their form depending on the distance H . At $H/d = 6$ and higher, there is a maximum of fluctuations near the stagnation point; at $H/d = 8$ its value attains 50% of the maximum value of friction. Away from the stagnation point the level of fluctuations decreases monotonically. Similarity between the profiles of the average and fluctuational values of friction for $H/d > 6$ is observed over the entire range of Reynolds numbers investigated.

The change in the maximum values of the friction factor on the barrier depending on the Reynolds number is approximated with good accuracy by the power law $C_{fm} = A(H/d) \cdot Re^{-n}$, where the exponent n for $H/d > 6$ lies within the range of 0.4–0.45, but for small values of H , when the level of turbulence in the neighborhood of the stagnation point is low, it approaches 0.5. This corresponds exactly to the value figuring in the theoretical model based on the momentum equation for an axisymmetric laminar boundary layer and confirms the equivalence of the two techniques used for calibrating the friction transducers. We note, however, that the power-law relation for the maximum friction factor at small distances between the nozzle and the barrier is valid only in the range of sufficiently high Reynolds numbers, i.e., in the self-similarity region. When $Re < 3 \cdot 10^4$ and $H/d < 5$, the change in C_{fm} has a nonmonotonic character.

For small values of H/d , a second maximum was already observed in the distributions of friction [2, 11, 12] and of heat and mass transfer coefficients [4, 20, 21]. Some authors explain its appearance by the transition of flow in the boundary layer on the barrier from a laminar to a turbulent regime. However, as follows from Fig. 2, in the region of regime nonself-similarity the position of the second friction maximum and of the maximum of fluctuations on an increase in the Reynolds number does not change, but according to [4] the maximum of heat transfer shifts downstream. If the reason for such nonmonotonicity in the distributions is the laminar-turbulent transition, the shift of both maxima with an increase in Re should occur in the direction of the stagnation point.

A similar picture of the change in the surface friction factor is observed in turbulent boundary layers under conditions of the transition from a negative to a positive pressure gradient [22]. In the present case a similar situation is observed. Precision measurements of static pressure on the barrier showed that at $r/d \approx 1.4-1.5$ the monotonic decrease of pressure with distance from the stagnation point is replaced by its insignificant increase, and at the point $r/d \approx 1.7$ a local maximum appears in the distribution of pressure. Its value comes to only 3% of the maximum value of pressure at the stagnation point. The minimum pressure preceding this maximum is approximately equal to $0.025P_0$.

The distributions of the root-mean-square pressure fluctuations are similar to the distributions of friction fluctuations (Fig. 4). For small values of H we also observe a maximum at $r/d \approx 1.7$ (Fig. 4a); for $H/d > 6$ the level of pressure fluctuations decreases monotonically with an increase in the distance from the stagnation point (Fig. 4b). The character of these dependences, as well as the presence of distinct maxima in the distributions of the spectral density of friction fluctuations obtained by the electrodiffusion method, confirms the hypothesis about

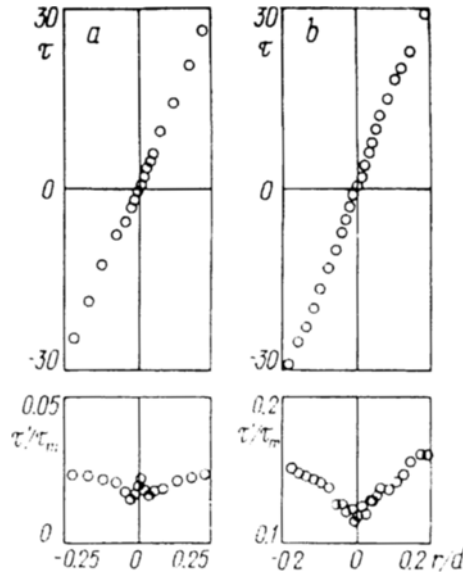


Fig. 5. Distributions of average and fluctuational values of friction on wall in vicinity of stagnation point, $d = 10$ mm, $Re = 41,600$: a) $H/d = 2$; b) $H/d = 5$.

the determining effect of quasiperiodic coherent structures on flow [6, 23]. According to an approach intensely developed in recent years, large-scale eddy structures make the main contribution to the enhancement of transfer processes. The small-amplitude waves of vorticity formed near the edges of the nozzle due to shear instability move farther downstream, grow, and are arranged in pairs. In a number of cases, the mechanisms of paired and of so-called collective interaction [23] caused by acoustic or hydrodynamic feedback explain the majority of the features of flow in impact jets.

Thus, the transition to highly turbulent flow on a barrier at $r/d \approx 1.7$ seems to be caused not by the development of instabilities formed in the boundary layer but by the interaction of large-scale structures with the plane. At the same time the smooth change in the hydrodynamic characteristics of flow at large values of H is explained by the disintegration of coherent vortices in the zone of the main portion of the jet.

The use of double electrochemical friction transducers made it possible to carry out measurements in the immediate vicinity of the stagnation point and to fix experimentally the point with a zero value of friction. The electrodiffusion method has certain limitations, in that in the case of small values of friction on a wall ($\tau \rightarrow 0$) it is necessary, when solving the diffusion equation, to take into account the quadratic term in the expansion of velocity near the wall:

$$u = \frac{\tau}{\mu} y + \frac{1}{2\mu} \frac{dP}{dx} y^2,$$

which is determined by the instantaneous value of the longitudinal pressure gradient. This considerably complicates the calculation of the dependence for the mass transfer coefficient. But actually the situation is such that in the overwhelming majority of types of flow there is a nonstationary separation or attachment of flow, when the friction is equal to zero only on the average but at each time instant it has a finite value with a positive or negative sign. This relates equally to the stagnation point of an impact jet, where even at relatively small Reynolds numbers and distances H the instantaneous friction modulus has a finite value. In all of the previous investigations, friction was measured by single-component transducers; this led inevitably to an error in the determination of its value in the vicinity of the stagnation point. The error increased with an increase in H , as well as in the absolute level of the alternating-sign fluctuations of flow.

The mean and fluctuational values of friction measured by a double electrodiffusion transducer in the vicinity of the stagnation point are presented in Fig. 5 for different values of H . The measuring system made it

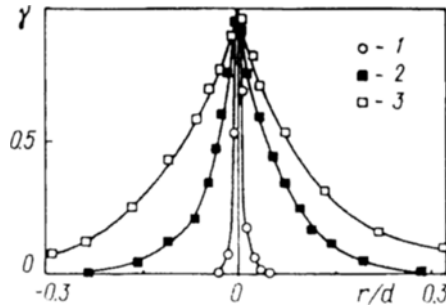


Fig. 6. Coefficient of reverse flows in vicinity of stagnation point, $d = 10$ mm, $Re = 41,600$: 1) $H/d = 2$; 2) 6; 3) 8.

possible to record the value of friction with its sign at each time instant. The signals were averaged separately for each sign. The mean value of friction was determined in this case by the simple addition of components. Using this procedure, we also obtained dependences for the so-called coefficient of reverse flow γ , which is the ratio of the times of existence of forward and reverse flows. The distributions of the coefficient γ in the vicinity of the stagnation point of an axisymmetric impact jet are given in Fig. 6. The dependences have a characteristic form with a sharp vertex and a smooth change with an increase in r . The dimensions of the region of reverse flows change sharply with an increase in H : from fractions of a millimeter at $H/d = 2$ to several millimeters at $H/d = 8$. Analysis of the dependences for the coefficient γ made it possible to generalize the data on the following basis. It was found that the half-width of the region of reverse flows in the vicinity of the stagnation point correlates rather well with only one parameter, i.e., with the absolute level of turbulent fluctuations of friction. On this basis, we analyzed the dependences for the ratio of these quantities. The complex obtained remains constant on two different levels corresponding to the cases in which the jet impinges on the barrier within the limits of its initial ($2 < H/d < 4$) and main ($H/d > 5$) portions.

Thus, in the present work for the first time we carried out systematic investigations of the fields of average and fluctuating velocity components, shear stress, and pressure on the wall for of an axisymmetric immersed jet. We determined the regime and geometric ranges of self-similarity in the distributions of hydrodynamic parameters. A weak effect of the conditions of jet outflow on the hydrodynamic characteristics of flow for well shaped converging nozzles is shown. For the first time the values of friction on the barrier in the immediate vicinity of the stagnation point were measured and the regions of the existence of alternating-sign flow near the wall were determined.

NOTATION

i , diffusion current, A; u , local velocity of fluid, m/sec; u_0 , mean flow rate of fluid at the nozzle outlet, m/sec; u_m , maximum velocity of fluid at the nozzle outlet, m/sec; τ , local shear stress on the wall, Pa; τ' , level of root-mean-square fluctuations of friction on the wall, Pa; H , distance from the nozzle tip to the barrier, m; d , nozzle diameter, m; P , static pressure on the wall, Pa; P' , level of root-mean-square fluctuations of pressure on the wall, Pa; x , longitudinal coordinate along the nozzle axis, m; r , radial coordinate along the barrier, m; y , transverse coordinate in the boundary layer on the barrier, m; $b_{0.5}$, half-width of the pressure profile, m; $Re = u_0 d / \nu$, where ν is the coefficient of kinematic viscosity, m^2/sec ; μ , coefficient of dynamic viscosity, $kg/m \cdot sec$; ρ , density of fluid, kg/m^3 ; $T_{um} = u' / u_0$, degree of turbulence at the nozzle outlet; $C_f = \tau / (\rho u_0^2 / 2)$, friction factor; $C_{fm} = \tau_m' / (\rho u_0^2 / 2)$, maximum friction factor on the barrier; $\gamma = [t(+)/t(-)]^{sign \tau}$, coefficient of reverse flows; $t(+)$, $t(-)$, times of existence of forward and reverse flows.

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